A21 Further Maths

Partial Fractions

*knowledge of A-level Maths A21 Partial Fractions is assumed.

The process of taking a single fraction and breaking it up into the sum (or difference) of 2 or more fractions is known as splitting an expression into partial fractions.

Note:- If the degree of the numerator is greater than or equal to the degree of the denominator you must first divide the numerator by the denominator.

Quadratic factors in the denominator

For a fraction that has a non-reducible quadratic factor on the denominator and where the degree of the denominator exceeds that of the numerator e.g.

$$\frac{x^2 - 5x + 1}{(x^2 + 1)(x - 2)}$$

then the partial fractions are of the form:-

$$\frac{Ax+B}{(x^2+1)} + \frac{C}{(x-2)}$$ where A,B and C are constants.

Example

Express $\frac{5x^2+4x+4}{(x+2)(x^2+4)}$ in partial fractions.

Solution

let $\frac{5x^2+4x+4}{(x+2)(x^2+4)} \equiv \frac{A}{x+2} + \frac{Bx+C}{x^2+4}$

$5x^2+4x+4 = A(x^2+4) + (Bx+C)(x+2)$

Let $x=-2$ $\quad 20-8+4 = A(8)$

$16 = 8A$

$A=2$

equate coeffs x^2 $\quad 5 = A + B$

$\therefore B=3$

equate coeff x $\quad 4 = 2B + C$

$4 = 6 + C$

$C=-2$

ans: $\frac{5x^2+4x+4}{(x+2)(x^2+4)} \equiv \frac{2}{x+2} + \frac{3x-2}{(x^2+4)}$

Note:- Remember to check that the denominator is completely factorised before attempting to put in partial fractions.

Example

Express $\frac{-2x-1}{(x^2-3x+2)(x^2-x+3)}$ in partial fractions.

Solution

$x^2-3x+2=(x-1)(x-2)$

Let

$\frac{-2x-1}{(x-1)(x-2)(x^2-x+3)} \equiv \frac{A}{x-1}+\frac{B}{x-2}+\frac{Cx+D}{x^2-x+3}$

$\therefore -2x-1=A(x-2)(x^2-x+3)+B(x-1)(x^2-x+3)+(Cx+D)(x-1)(x-2)$

Let $x=1$, $-3=A(-1)(3)$, $A=1$

Let $x=2$, $-5=B(1)(5)$, $B=-1$

equate x^3: $0=A+B+C$, $0=1-1+C$, $C=0$

let $x=0$: $-1=A(-2)(3)+B(-1)(3)+D(-1)(-2)$

$-1=-6A-3B+2D$

$-1=-6+3+2D$

$-1+3=2D$

$D=1$

$\therefore \frac{-2x-1}{(x^2-3x+2)(x^2-x+3)} \equiv \frac{1}{x-1}-\frac{1}{x-2}+\frac{1}{x^2-x+3}$ ✓

Note:-

$$x^3-1=(x-1)(x^2+x+1)$$

$$x^3+1=(x+1)(x^2-x+1)$$

$$x^3-y^3=(x-y)(x^2+xy+y^2)$$

$$x^3+y^3=(x+y)(x^2-xy+y^2)$$

*P2 book Ex1D Q6,7,8,9,10,12,23,24,28,29,31,33,35