Repeated integration by parts/reduction formula

-P2 book P289 Ex9D Q1,3,5,6,9, 14-16,18,19

Exercise 9D

Use integration by parts to find:

1 $\int xe^{-x}\,dx$ 2 $\int xe^{3x}\,dx$ 3 $\int x\sin x\,dx$

4 $\int x\ln x\,dx$ 5 $\int \ln(x-1)\,dx$ 6 $\int x\cos 3x\,dx$

7 $\int x(x-1)^4\,dx$ 8 $\int x\sqrt{(x-1)}\,dx$ 9 $\int x^2e^{x}\,dx$

10 $\int x^2\cos x\,dx$ 11 $\int x^2e^{-x}\,dx$ 12 $\int x^3\ln x\,dx$

Evaluate each of the following definite integrals:

13 $\int_0^{\pi} x\sin x\,dx$ 14 $\int_0^{\frac{\pi}{2}} x\cos\frac{1}{2}x\,dx$ 15 $\int_1^{e} x^2\ln x\,dx$

16 $\int_0^{1} x(x-1)^3\,dx$ 17 $\int_0^{2} (x-1)(x+1)^3\,dx$

18 $\int_1^{e} \frac{\ln x}{x^4}\,dx$ 19 $\int_1^{e} (\ln x)^2\,dx$ 20 $\int_0^{\frac{\pi}{2}} e^{x}\sin x\,dx$

ANSWERS

Exercise 9D

[The constant of integration is omitted in indefinite integration.]

1 $-e^{-x}(x+1)$ **2** $\frac{1}{3}xe^{3x}-\frac{1}{9}e^{3x}$

3 $-x\cos x+\sin x$

4 $\frac{x^2}{2}\ln|x|-\frac{x^2}{4}$

5 $x\ln|x-1|-x-\ln|x-1|$

6 $\frac{1}{3}x\sin 3x+\frac{1}{9}\cos 3x$

7 $\frac{(5x+1)(x-1))^5}{30}$ **8** $\frac{2}{15}(3x+2)(x-1)^{\frac{3}{2}}$

9 $e^{x}(x^2-2x+2)$

10 $x^2\sin x+2x\cos x-2\sin x$

11 $-e^{-x}(x^2+2x+2)$

12 $\frac{x^4}{16}(4\ln x-1)$ **13** π

14 $\frac{\pi}{\sqrt{2}}+\frac{4}{\sqrt{2}}-4$ **15** $\frac{2}{9}e^3+\frac{1}{9}$ **16** $-\frac{1}{20}$

17 8.4 **18** $\frac{1}{9}(1-4e^{-3})$ **19** $e-2$

20 $\frac{1}{2}(e^{\frac{\pi}{2}}+1)$

-P4 book page 202 Ex5A Q 1,2,3,5,7,9-13 leave Q4 and 6 to the end

Exercise 5A

1 Given that $I_n = \int \cos^n x \, dx$, show that, for $n \geqslant 2$,

$$nI_n = \cos^{n-1} x \sin x + (n-1)I_{n-2}$$

Hence evaluate I_8 for $I_n = \int_0^{\frac{\pi}{2}} \cos^n x \, dx$, and I_9 for $\int_0^{\pi} \cos^n x \, dx$.

2 Given that $I_n = \int (\ln x)^n dx$, show that, for $n \geqslant 1$,

$$I_n = x(\ln x)^n - nI_{n-1}$$

Hence evaluate $\int_1^2 (\ln x)^3 dx$.

3 Given that $I_n = \int_0^{2\pi} \sin^n x \, dx$, find a reduction relation between I_n and I_{n-2} for $n \geqslant 2$. Hence find I_5 and I_6.

4 Evaluate (a) $\int_0^{\frac{\pi}{2}} \sin^3 \theta \cos^4 \theta \, d\theta$ (b) $\int_0^{\frac{\pi}{2}} \sin^2 \theta \cos^6 \theta \, d\theta$.

5 Use the substitution $x = \sin t$ and an appropriate reduction formula to evaluate

$$\int_0^1 x^6 (1 - x^2)^{\frac{1}{2}} \, dx$$

6 Use the substitution $x = \sin^2 t$ and an appropriate reduction formula to evaluate

$$\int_0^1 x(1 - x)^{\frac{3}{2}} \, dx$$

Check your answer by using another method.

7 Using an appropriate reduction formula, evaluate, in terms of e, the integral $\int_0^1 x^5 e^x \, dx$.

8 Given that $I_n = \int \cosh^n x \, dx$, show that, for $n \geqslant 2$,

$$nI_n = \cosh^{n-1} x \sinh x + (n-1)I_{n-2}$$

Hence find $\int \cosh^4 x \, dx$.

9 Given that $I_n = \int_0^1 x^n e^{-x} dx$, show that, for $n \geqslant 1$,

$$I_n = nI_{n-1} - e^{-1}$$

Hence evaluate $\int_0^1 x^6 e^{-x} dx$.

10 Given that $I_n = \int_0^{\frac{\pi}{3}} \sec^n x \, dx$ show that, for $n \geqslant 2$,

$$I_n = \frac{2^{n-2}}{n-1}\sqrt{3} + \frac{n-2}{n-1} I_{n-2}$$

Hence evaluate I_7.

11 Given that $I_n = \int_0^1 \frac{1}{(1+x^2)^n} dx$, show that, for $n \geqslant 2$,

$$2(n-1)I_n = 2^{1-n} + (2n-3)I_{n-1}$$

Hence find I_3.

12 Given that $I_n = \int \frac{\sin nx}{\sin x} dx$, $n \geqslant 2$,

show that $$I_n = \frac{2 \sin (n-1)x}{n-1} + I_{n-2}$$

Hence evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin 4x}{\sin x} dx$ and check your answer by using another method.

13 Given that $I_n = \int_0^{\frac{\pi}{2}} x^n \sin x \, dx$, show that, for $n \geqslant 2$,

$$I_n = n\left(\tfrac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2}$$

Hence evaluate I_3.

14 By setting up an appropriate reduction formula for $\int x^n \sinh x \, dx$ and applying the formula show that

$$\int x^5 \sinh x \, dx = \cosh x\,(x^5 + 20x^3 + 120x) - \sinh x\,(5x^4 + 60x^2 + 120) + C$$

where C is a constant.

15 Given that $I_n = \int_0^2 (4 - x^2)^n \, dx$, show that, for $n \geqslant 1$,

$$I_n = \frac{8n}{2n+1} I_{n-1}$$

Evaluate I_4.

ANSWERS

Exercise 5A

1 $\frac{35}{256}\pi$, $\frac{128}{315}$

2 $2(\ln 2)^3 - 6(\ln 2)^2 + 12 \ln 2 - 6$

3 $I_n = \dfrac{n-1}{n} I_{n-2}$, $I_5 = 0$, $I_6 = \frac{5}{8}\pi$

4 (a) $\frac{2}{35}$ (b) $\frac{5}{256}\pi$

5 $\frac{5}{256}\pi$

6 $\frac{4}{35}$

7 $120 - 44e$

8 $\frac{1}{4}\cosh^3 x \sinh x + \frac{3}{8}\cosh x \sinh x + \frac{3}{8}x + C$

9 $720 - 1957e^{-1}$

10 $\frac{61}{8}\sqrt{3} + \frac{5}{16}\ln(2 + \sqrt{3})$

11 $\frac{1}{4} + \frac{3}{32}\pi$

12 $\frac{4}{3}(1 - \sqrt{2})$

13 $\frac{3}{4}(\pi^2 - 8)$

15 $\frac{65\,536}{315}$