Differentiation of arcsin, arcos, arctan

-P3 book P52 Ex3C Q7-15

Exercise 3C

1 Find, in radians in terms of π, the value of:

(a) $\arcsin 1$ (b) $\arcsin(-\frac{\sqrt{3}}{2})$ (c) $\arccos\frac{\sqrt{3}}{2}$

(d) $\arccos 0$ (e) $\arctan(-\sqrt{3})$ (f) $\arctan(2+\sqrt{3})$

2 Giving your answer in radians to 2 decimal places, find the value of:

(a) $\arcsin(0.75)$ (b) $\arctan 7$ (c) $\arccos(-0.735)$

(d) $\arcsin(-0.993)$ (e) $\arccos(-0.111)$ (f) $\arctan(-0.352)$

3 Given that $y = \operatorname{cosec} x$, $-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$, $x \neq 0$, sketch the graphs of the curves $y = \operatorname{cosec} x$ and $y = \operatorname{arccosec} x$, where $\operatorname{arccosec} x$ is the inverse function of $\operatorname{cosec} x$, $-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$, $x \neq 0$.

4 Given that $y = \cot x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, $x \neq 0$, sketch the graphs of the curves $y = \cot x$ and $y = \operatorname{arccot} x$, where $\operatorname{arccot} x$ is the inverse function of $\cot x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, $x \neq 0$.

5 Given that $y = \sec x$, $0 \leqslant x \leqslant \pi$, $x \neq \frac{\pi}{2}$, sketch the graphs of the curves $y = \sec x$ and $y = \operatorname{arcsec} x$, where $\operatorname{arcsec} x$ is the inverse function of $\sec x$, $0 \leqslant x \leqslant \pi$, $x \neq \frac{\pi}{2}$.

6 Find the smallest positive value of x for which

(a) $\tan 2x = \sqrt{3}$

(b) $\sin(2x - 3) = \frac{1}{2}$

(c) $\sin x = \cos(\arctan 1)$

7 Differentiate with respect to x:

(a) $\arcsin 3x$ (b) $(\arcsin x)^2$ (c) $\arcsin\left(\frac{1}{x}\right)$

8 Differentiate with respect to x:

(a) $\arccos\left(\frac{x}{2}\right)$ (b) $\arccos(3x^2)$ (c) $\arccos\left(\frac{1}{x+2}\right)$

9 Differentiate with respect to x:

(a) $\arctan(2x)$ (b) $e^{\arctan x}$ (c) $\arctan(\ln x)$

10 Differentiate with respect to x:

(a) $x \arcsin x$ (b) $e^x \arccos x$ (c) $\dfrac{e^x}{\arctan x}$

(d) $\arctan\left(\dfrac{1-\sqrt{x}}{1+\sqrt{x}}\right)$ (e) $\operatorname{arcsec} x^2$

11 Given that $y = \arcsin x$, show that:

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$$

12 Given that $y = x - \arctan x$, show that:

$$\frac{d^2y}{dx^2} - 2x\left(1 - \frac{dy}{dx}\right)^2 = 0$$

13 Find (a) $\frac{d}{dx}(\operatorname{arcsec} x)$ (b) $\frac{d}{dx}(\operatorname{arccot} x)$.

14 Find an equation at the point where $x = \frac{2}{\sqrt{3}}$ of the tangent to the curve $y = \operatorname{arccosec} x$.

15 Given that k is a positive constant, differentiate

(a) $\arccos \frac{x}{k}$ (b) $\arcsin \frac{k}{x}$ (c) $\arctan \frac{k}{x}$

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Exercise 3C

1 (a) $\frac{\pi}{2}$ (b) $-\frac{\pi}{3}$ (c) $\frac{\pi}{6}$
(d) $\frac{\pi}{2}$ (e) $-\frac{\pi}{3}$ (f) $\frac{5\pi}{12}$

2 (a) 0.85 (b) 1.43 (c) 2.40
(d) −1.45 (e) 1.68 (f) −0.34

6 (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{12} + \frac{3}{2}$ (c) $\frac{\pi}{4}$

7 (a) $\frac{3}{\sqrt{(1-9x^2)}}$ (b) $\frac{2\arcsin x}{\sqrt{(1-x^2)}}$
(c) $\frac{-1}{x\sqrt{(x^2-1)}}$

8 (a) $\frac{-1}{\sqrt{(4-x^2)}}$ (b) $\frac{-6x}{\sqrt{(1-9x^4)}}$
(c) $\frac{-1}{(x+2)\sqrt{(x^2+4x+3)}}$

9 (a) $\frac{2}{1+4x^2}$ (b) $\frac{e^{\arctan x}}{1+x^2}$
(c) $\frac{1}{x[1+(\ln x)^2]}$

10 (a) $\arcsin x + \frac{x}{\sqrt{(1-x^2)}}$
(b) $e^x \arccos x - \frac{e^x}{\sqrt{(1-x^2)}}$
(c) $\frac{e^x \arctan x - \frac{e^x}{1+x^2}}{(\arctan x)^2}$
(d) $\frac{-1}{2(1+x)\sqrt{x}}$ (e) $\frac{2}{x\sqrt{(x^4-1)}}$

13 (a) $\frac{1}{x\sqrt{(x^2-1)}}$ (b) $\frac{-1}{1+x^2}$

14 $y - \frac{\pi}{3} = -\frac{3}{2}\left(x - \frac{2}{\sqrt{3}}\right)$

15 (a) $\frac{-1}{\sqrt{(k^2-x^2)}}$ (b) $\frac{-k}{x\sqrt{(x^2-k^2)}}$
(c) $\frac{-k}{x^2+k^2}$

Integration of $\frac{1}{a^2+x^2}$ and $\frac{1}{\sqrt{a^2-x^2}}$

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Exercise 15H

Differentiate the following with respect to x

1. $\sin^{-1} x$ **2.** $\tan^{-1} \frac{x}{a}$ **3.** $\sin^{-1} \frac{x}{4}$

4. $\cos^{-1} 3x$ **5.** $\tan^{-1} 4x$ **6.** $\sin^{-1} 6x$

7. $\sin^{-1} (2x - 1)$ **8.** $\tan^{-1} (1 - 3x)$ **9.** $\sin^{-1} (x^2 - 1)$

10. $x \sin^{-1} x$ **11.** $x \tan^{-1} x$ **12.** $(x^2 + 1) \tan^{-1} x$

Find the following indefinite integrals

13. $\int \frac{1}{\sqrt{(4 - x^2)}} dx$ **14.** $\int \frac{1}{\sqrt{(16 - x^2)}} dx$ **15.** $\int \frac{3}{9 + x^2} dx$

16. $\int \frac{1}{25 + x^2} dx$ **17.** $\int \frac{1}{\sqrt{(49 - x^2)}} dx$ **18.** $\int \frac{1}{49 + x^2} dx$

19. $\int \frac{1}{\sqrt{(25 - x^2)}} dx$ **20.** $\int \frac{2}{100 + x^2} dx$

Evaluate the following definite integrals (leave π in your answers).

21. $\int_{-3}^{3} \frac{1}{\sqrt{(36 - x^2)}} dx$ **22.** $\int_{-2}^{2} \frac{1}{4 + x^2} dx$ **23.** $\int_{-3}^{3} \frac{\sqrt{3}}{x^2 + 3} dx$

24. $\int_{0}^{3\sqrt{2}} \frac{1}{\sqrt{(3 - x^2)}} dx$

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1. $\frac{1}{\sqrt{(1 - x^2)}}$ **2.** $\frac{a}{a^2 + x^2}$ **3.** $\frac{1}{\sqrt{(16 - x^2)}}$ **4.** $\frac{-3}{\sqrt{(1 - 9x^2)}}$ **5.** $\frac{4}{1 + 16x^2}$

6. $\frac{6}{\sqrt{(1 - 36x^2)}}$ **7.** $\frac{1}{\sqrt{(x(1 - x))}}$ **8.** $\frac{-3}{2 - 6x + 9x^2}$ **9.** $\frac{2}{\sqrt{(2 - x^2)}}$

10. $\sin^{-1} x + \frac{x}{\sqrt{(1 - x^2)}}$ **11.** $\tan^{-1} x + \frac{x}{1 + x^2}$ **12.** $2x \tan^{-1} x + 1$

13. $\sin^{-1}\left(\frac{x}{2}\right) + c$ **14.** $\sin^{-1}\left(\frac{x}{4}\right) + c$ **15.** $\tan^{-1}\left(\frac{x}{3}\right) + c$ **16.** $\frac{1}{5}\tan^{-1}\left(\frac{x}{5}\right) + c$

17. $\sin^{-1}\left(\frac{x}{7}\right) + c$ **18.** $\frac{1}{7}\tan^{-1}\left(\frac{x}{7}\right) + c$ **19.** $\sin^{-1}\left(\frac{x}{5}\right) + c$ **20.** $\frac{1}{5}\tan^{-1}\left(\frac{x}{10}\right) + c$

21. $\frac{\pi}{3}$ **22.** $\frac{\pi}{4}$ **23.** $\frac{7}{12}\pi$ **24.** $\frac{\pi}{3}$