Summation of Series

- P3 book P15 Ex2A Q1,3,4,7,9,10

Exercise 2A

In each case, use the identity given to find the sum to n terms of the given series.

	Identity	Series
1	$\frac{1}{r(r+1)} \equiv \frac{1}{r} - \frac{1}{r+1}$	$\sum_{r=1}^{n} \frac{1}{r(r+1)}$
2	$2r+1 \equiv (r+1)^2 - r^2$	$\sum_{r=1}^{n} (2r+1)$
3	$\frac{2}{4r^2-1} \equiv \frac{1}{2r-1} - \frac{1}{2r+1}$	$\sum_{r=1}^{n} \frac{1}{4r^2-1}$
4	$r^2(r+1) - (r-1)^2(r) \equiv 3r^2 - r$	$\sum_{r=1}^{n} r(3r-1)$
5	$\frac{r}{r+1} - \frac{r-1}{r} \equiv \frac{1}{r(r+1)}$	$\sum_{r=1}^{n} \frac{1}{r(r+1)}$
6	$4r(r+1)(r+2) \equiv r(r+1)(r+2)(r+3) - (r-1)(r)(r+1)(r+2)$	$\sum_{r=1}^{n} r(r+1)(r+2)$
7	$\frac{2}{r(r+1)(r+2)} \equiv \frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}$	$\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)}$
8	$\frac{2r+1}{r^2(r+1)^2} \equiv \frac{1}{r^2} - \frac{1}{(r+1)^2}$	$\sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2}$

9 Use the identity $(r+1)^3 - r^3 \equiv 3r^2 + 3r + 1$ to find

$$\sum_{r=1}^{n} r(r+1).$$

10 Show that $\frac{1}{r!} - \frac{1}{(r+1)!} \equiv \frac{r}{(r+1)!}$. Hence find $\sum_{r=1}^{n} \frac{r}{(r+1)!}$.

- P3 book P18 Ex2B Q1-3, 5-7, 9-11, 13, 14, 17, 19

Exercise 2B

Evaluate:

1 $\sum_{r=1}^{13} r^2$

2 $\sum_{r=4}^{11} r^3$

3 $\sum_{r=11}^{24} r(r+1)$

4 $\sum_{r=1}^{19} r(r+4)$

5 $\sum_{r=1}^{20} \frac{1}{r(r+1)}$

6 $\sum_{r=3}^{16} (r+2)^3$

7 $\sum_{r=1}^{14} \left(\frac{3}{4}\right)^r$

8 $\sum_{r=1}^{20} \frac{1}{(r+3)(r+6)}$

9 $\sum_{r=4}^{16} (2r-1)^3$

10 $\sum_{r=3}^{23} r(r+1)(r+2)$

11 Show that $\sum_{r=1}^{n} (2r-1)^2 \equiv \frac{1}{3}n(4n^2-1)$.

12 Show that $\sum_{r=1}^{n} r(2+r) \equiv \frac{1}{6}n(n+1)(2n+7)$.

13 Find $\sum_{r=1}^{20} \frac{1}{4r^2-1}$.

14 Find $\sum_{r=n}^{2n} r^2$.

15 Given that $f(r) \equiv \frac{1}{r(r+1)}$, show that

$$f(r) - f(r+1) \equiv \frac{2}{r(r+1)(r+2)}$$

Hence find $\sum_{r=5}^{25} \frac{1}{r(r+1)(r+2)}$.

16 Prove that $\sum_{r=1}^{n} \frac{1}{(r+1)(r+2)} = \frac{n}{2(n+2)}$.

17 Find the sum of all even numbers between 2 and 200 inclusive, excluding those which are multiples of 3.

18 Find $\sum_{r=1}^{100} 2r^2 - \sum_{r=1}^{200} r^2$.

19 Find the sum of the series

$$1^2 - 2^2 + 3^2 - 4^2 + \cdots - (2n)^2$$

20 Given that $u_r = r(2r+1) + 2^{r+2}$, find $\sum_{r=1}^{n} u_r$.

ANSWERS

Exercise 2A

1 $1-\dfrac{1}{n+1}$ **2** n^2+2n

3 $\dfrac{n}{2n+1}$ **4** $n^2(n+1)$

5 $\dfrac{n}{n+1}$

6 $\frac{1}{4}n(n+1)(n+2)(n+3)$

7 $\dfrac{n(n+3)}{4(n+1)(n+2)}$ **8** $\dfrac{n(n+2)}{(n+1)^2}$

9 $\frac{1}{3}n(n+1)(n+2)$ **10** $1-\dfrac{1}{(n+1)!}$

Exercise 2B

1 819 **2** 4320 **3** 4760

4 3230 **5** $\frac{20}{21}$ **6** 29 141

7 2.95 (3 s.f.) **8** 0.1655 (4 d.p.)

9 130 663 **10** 89 670 **13** $\frac{20}{41}$

14 $\dfrac{n}{6}(n+1)(14n+1)$ **15** $\frac{28}{1755}$

17 6734 **18** $-2\,010\,000$

19 $-n(2n+1)$

20 $\dfrac{n}{6}(n+1)(4n+5)+2^{n+3}-8$