A-level Further Maths A21

Induction

A theorem thought to be true for all values of the positive integer n, can be proved by showing that:-

(i) If it is true for n=k, then it is also true for n=k+1.

and

(ii) It is true for some small value of n such as n=1 (or perhaps n=2 or n=3)

If you prove both (i) and (ii) then you have shown that the theorem is true at the start (usually n=1) and it ids true for n=1+1 and n=2+1 and n=3+1 and so on for all integer values of n following on after the valid starting value (usually n=1).

Example 1 Use the method of mathematical induction to prove:-

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

where n is a positive integer.

Proof

Example 2 Use the method of mathematical induction to prove that the expression:-

$$3^{2n} + 7$$

Is divisible by 8 for all positive integers n.

Proof (Method 1)

**See other method too

Proof (Method 2)

Example Given that n is an integer, which is greater than 3, show that

$$n! > 2^n$$

Proof

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