<u>Summation of Finite Series Using The Method Of Differences</u>

$$\sum_{r=1}^{n} r = 1 + 2 + 3 + \cdots + n \ (frontwards)$$

$$\sum_{r=1}^{n} r = n + (n-1) + (n-2) \ldots 3 + 2 + 1 \ (backwards)$$

Adding:-

$$2\sum_{r=1}^{n} r = (n+1) + (n+1) + (n+1) + \cdots (n+1) + (n+1) + (n+1)$$

(n terms)

$$2\sum_{r=1}^{n} r = n(n+1)$$

Result 1:-

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$

Note:- Here is another way you could sum the series $1 + 2 + 3 + \cdots + n$.

Consider the identity

$$2r \equiv r(r+1) - (r-1)r$$

Taking successive values 1,2,3,….,n for r, we get:-

This method is called summing a series by the method of difference.

Genereally if it is possible to find a function f® such that the rth term u_r of a series can be expressed as

$u_r = f(r+1) - f(r)$, then it is easy to find

$$\sum_{r=1}^{n} u_r$$

We have for r=1,2,3,…,n

$$u_1 = f(2) - f(1)$$

$$u_2 = f(3) - f(2)$$

$$u_3 = f(4) - f(3)$$

.. .. ..

.. .. ..

$$u_n = f(n+1) - f(n)$$

Adding:-

$$\sum_{r=1}^{n} u_r = f(n+1) - f(1)$$

because all the other terms on R.H.S. cancel out.

Example 1:-Find

$$\sum_{r=1}^{n} r^2$$

Consider the identity

$$24r^2 + 2 \equiv (2r+1)^3 - (2r-1)^3$$

And take r=1,2,3,…,n.

Solution

Result 2:-

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

Example 2:-Find

$$\sum_{r=1}^{n} r^3$$

Consider the identity

$$4r^3 \equiv r^2(r+1)^2 - (r-1)^2r^2$$

And take r=1,2,3,…,n.

Solution

Result 3:-

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

Note:- Since

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$

Then

$$\sum_{r=1}^{n} r^3 = \left(\sum_{r=1}^{n} r\right)^2$$

Example 3:- Find

$$\sum_{r=1}^{n} r(r+1)$$

Consider the identity

$$3r(r+1) \equiv r(r+1)(r+2) - (r-1)(r)(r+1)$$

And take r=1,2,3,…,n.

Solution

Results for the sigma notation:-

1.

$$\sum_{r=1}^{n} af(r) = a\sum_{r=1}^{n} f(r)$$

Proof:-

$$\sum_{r=1}^{n} af(r) = af(1) + af(2) + af(3) + \cdots + af(n)$$

$$\sum_{r=1}^{n} af(r) = a[f(1) + f(2) + f(3) + \cdots + f(n)]$$

$$\therefore \sum_{r=1}^{n} af(r) = a\sum_{r=1}^{n} f(r)$$

2.

$$\sum_{r=1}^{n} f(r) + g(r) = \sum_{r=1}^{n} f(r) + \sum_{r=1}^{n} g(r)$$

Proof:-

$$\sum_{r=1}^{n} f(r) + g(r) = f(1) + g(1) + f(2) + g(2) + \cdots f(n) + g(n)$$

$$\sum_{r=1}^{n} f(r) + g(r) = [f(1) + f(2) + \cdots + f(n)] + [g(1) + g(2) + \cdots + g(n)]$$

$$\sum_{r=1}^{n} f(r) + g(r) = \sum_{r=1}^{n} f(r) + \sum_{r=1}^{n} g(r)$$

Questions: P3 book Page 15 Exercise 2A Q1,3,4,7,9,10

Telescoping Series

Example:- Find the value of

$$\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)}$$

Solution

The above is an example of a telescoping series, since the terms of S_n, other than the first and last, cancel out in pairs.

Summation of Finite Series Using Standard Results

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$$

Example:- Find

$(a.)\sum_{r=7}^{20} r^2$

$(b.)\sum_{r=12}^{25} r^3$

Solution

Example:- Show

$$\sum_{r=1}^{n} r(r+1) = \frac{1}{3}n(n+1)(n+2)$$

Solution

Example:- Find the following in terms of n.

$$\sum_{r=1}^{n} 6r^2 + 2^r$$

Solution

***Questions P3 book page 18 Ex2B Q1-3,5,-7,9-11,13,14,17,19