Complex Numbers Question Booklet for AS1

-P3 Book p166 Ex7A Q1ace,2ace,3ace,4,5ace,7ace,8ace,9ace,11,12 (*AS)

Exercise 7A

1 Express in terms of i:

(a) $\sqrt{(-64)}$ (b) $\sqrt{(-7)}$ (c) $\sqrt{16}-\sqrt{(-81)}$

(d) $3-\sqrt{(-25)}$ (e) $\sqrt{(-100)}-\sqrt{(-49)}$

2 Simplify:

(a) i^3 (b) i^7 (c) i^{-9}

(d) $i(2i-3i^3)$ (e) $(i+2i^2)(3-i)$

3 Write in the form $a+ib$, where $a, b \in \mathbb{R}$:

(a) $2i(5-2i)$ (b) $(2+i)^2$ (c) $(4-i)^5$

(d) $(1+2i)^2+(3-i)^3$ (e) $(1+i)^2-3(2-i)^3$

4 Find z^* given that $z =$

(a) $2+4i$ (b) $3-6i$ (c) $-5+2i$

(d) $-7-3i$ (e) $2i-4$ (f) 6

(g) $3i$ (h) $-3i+7$

5 Simplify:

(a) $(2+3i)+(4-7i)$ (b) $(-3+5i)+(-6-7i)$

(c) $(-7-10i)+(2-3i)$ (d) $(2+4i)-(3-6i)$

(e) $(-3+5i)-(-7+4i)$ (f) $(-9-6i)-(-8-9i)$

(g) $(6-3i)-(8-5i)$

6 Express in the form $a+ib$, where $a, b \in \mathbb{R}$:

(a) $(2+i)(3-i)$ (b) $(-3-4i)(2-7i)$

(c) $(5+2i)(-3+4i)$ (d) $(1-5i)^2$

(e) $(2-i)^3$ (f) $(1+i)(2-i)(i+3)$

(g) $i(3-7i)(2-i)$

7 Express in the form $a+ib$, where $a, b \in \mathbb{R}$:

(a) $\dfrac{2-7i}{1+2i}$ (b) $\dfrac{1+2i}{3-i}$ (c) $\dfrac{1+2i}{3+4i}$

(d) $\dfrac{1}{1+2i}$ (e) $\dfrac{2+3i}{2-3i}$ (f) $\dfrac{5+i}{i-3}$

(g) $\dfrac{6}{4i-3}$ (h) $\dfrac{1}{(i+2)(1-2i)}$

8 Solve:

(a) $x^2 + 25 = 0$ (b) $x^3 + 64x = 0$

(c) $x^2 - 4x + 5 = 0$ (d) $x^2 + 6x + 10 = 0$

(e) $x^2 + 29 = 4x$ (f) $2x^2 + 3x + 7 = 0$

(g) $3x^2 + 2x + 1 = 0$ (h) $3x^2 - 2x + 2 = 0$

9 Express in the form $a + ib$, where $a, b \in \mathbb{R}$:

(a) $\dfrac{1}{1+2i} + \dfrac{1}{1-2i}$ (b) $\dfrac{1}{2+i} - \dfrac{1}{1+5i}$

(c) $5 - 4i + \dfrac{5}{3-4i}$

10 Given that $z = -1 + 3i$, express $z + \dfrac{2}{z}$ in the form $a + ib$, where $a, b \in \mathbb{R}$.

11 Given that $T = \dfrac{x - iy}{x + iy}$, where x, y, $T \in \mathbb{R}$, show that

$$\frac{1 + T^2}{2T} = \frac{x^2 - y^2}{x^2 + y^2}$$

12 Show that the complex number $\dfrac{2+3i}{5+i}$ can be expressed in the form $\lambda(1 + i)$, where λ is real.
State the value of λ.

Hence, or otherwise, show that $\left(\dfrac{2+3i}{5+i}\right)^4$ is real and determine its value. [L]

ANSWERS

Exercise 7A

1 (a) $8i$ (b) $i\sqrt{7}$ (c) $4-9i$
(d) $3-5i$ (e) $3i$

2 (a) $-i$ (b) $-i$ (c) $-i$
(d) -5 (e) $-5+5i$

3 (a) $4+10i$ (b) $3+4i$
(c) $404-1121i$ (d) $15-22i$
(e) $-6+35i$

4 (a) $2-4i$ (b) $3+6i$ (c) $-5-2i$
(d) $-7+3i$ (e) $-4-2i$ (f) 6
(g) $-3i$ (h) $7+3i$

5 (a) $6-4i$ (b) $-9-2i$
(c) $-5-13i$ (d) $-1+10i$
(e) $4+i$ (f) $-1+3i$
(g) $-2+2i$

6 (a) $7+i$ (b) $-34+34i$
(c) $-23+14i$ (d) $-24-10i$
(e) $2-11i$ (f) $8+6i$
(g) $17-i$

7 (a) $-\frac{1}{5}(12+11i)$ (b) $\frac{1}{10}(1+7i)$
(c) $\frac{1}{25}(11+2i)$ (d) $\frac{1}{5}(1-2i)$
(e) $\frac{1}{13}(-5+12i)$ (f) $-\frac{1}{5}(7+4i)$
(g) $-\frac{6}{25}(3+4i)$ (h) $\frac{1}{25}(4+3i)$

8 (a) $\pm 5i$ (b) $0, \pm 8i$ (c) $2 \pm i$
(d) $-3 \pm i$ (e) $2 \pm 5i$
(f) $\frac{1}{4}(-3 \pm i\sqrt{47})$
(g) $\frac{1}{3}(-1 \pm i\sqrt{2})$ (h) $\frac{1}{3}(1 \pm i\sqrt{5})$

9 (a) $\frac{2}{5}$ (b) $\frac{1}{130}(47-i)$ (c) $\frac{4}{5}(7-4i)$

10 $-\frac{6}{5}(1-2i)$

12 $\lambda = \frac{1}{2}; -\frac{1}{4}$

-P3 Book p179 Ex7C Q1ace,2ace,3,4,8 (*AS)

Exercise 7C

1 Find the square roots of:

(a) $5 + 12i$ (b) $7 - 24i$ (c) $3 - 4i$

(d) $-20i$ (e) $1 - i4\sqrt{3}$

2 Find the real numbers x and y given that:

(a) $x + 4y + xyi = 12 - 16i$

(b) $2x + (x - 2y)i = 18 - y - i$

(c) $3x + 2xi = 7 + 2y + (12 + 5y)i$

(d) $x - 7y + 8xi = 6y + (6y - 100)i$

(e) $2x - y + (y - 4)i = 0$

3 Given that $(1 + 5i)A - 2B = 3 + 7i$, find A and B if:

(a) A and B are real,

(b) A and B are conjugate complex numbers.

4 Given that $x, y \in \mathbb{R}$ and

$$(x + iy)(2 + i) = 3 - i$$

find x and y.

8 Given that

$$\frac{1}{x + iy} + \frac{1}{1 + 2i} = 1$$

where x and y are real, find x and y. [L]

ANSWERS

Exercise 7C

1 (a) $\pm(3 + 2i)$ (b) $\pm(4 - 3i)$ (c) $\pm(2 - i)$

(d) $\pm\sqrt{10}(1 - i)$ (e) $\pm(2 - i\sqrt{3})$

2 (a) $(-4, 4)$, $(16, -1)$

(b) $(7, 4)$ (c) $(1, -2)$

(d) $(-\frac{650}{49}, -\frac{50}{49})$ (e) $(2, 4)$

3 (a) $A = \frac{7}{5}$, $B = -\frac{4}{5}$

(b) $A = 2 - i$, $B = 2 + i$

4 $x = 1$, $y = -1$

5 (a) $p = 3$, $q = 1$ (b) $p = -\frac{2}{5}$, $q = \frac{6}{5}$

6 $x = -\frac{7}{25}$, $y = -\frac{24}{25}$ **7** $x = \frac{2}{13}$, $y = \frac{3}{13}$

8 $x = 1$, $y = -\frac{1}{2}$ **9** $a = 0$, $b = \pm 2$

-**Edexcel Book Ex1F Q1ace,2ace,3,4,8 (*AS)**

Exercise 1F

1 $f(z) = z^3 - 6z^2 + 21z - 26$

a Show that $f(2) = 0$. **(1 mark)**

b Hence solve $f(z) = 0$ completely. **(3 marks)**

2 $f(z) = 2z^3 + 5z^2 + 9z - 6$

a Show that $f\left(\frac{1}{2}\right) = 0$. **(1 mark)**

b Hence write $f(z)$ in the form $(2z - 1)(z^2 + bz + c)$, where b and c are real constants to be found. **(2 marks)**

c Use algebra to solve $f(z) = 0$ completely. **(2 marks)**

3 $g(z) = 2z^3 - 4z^2 - 5z - 3$

Given that $z = 3$ is a root of the equation $g(z) = 0$, solve $g(z) = 0$ completely. **(4 marks)**

4 $p(z) = z^3 + 4z^2 - 15z - 68$

Given that $z = -4 + i$ is a solution to the equation $p(z) = 0$,

a show that $z^2 + 8z + 17$ is a factor of $p(z)$. **(2 marks)**

b Hence solve $p(z) = 0$ completely. **(2 marks)**

5 $f(z) = z^3 + 9z^2 + 33z + 25$

Given that $f(z) = (z + 1)(z^2 + az + b)$, where a and b are real constants,

a find the value of a and the value of b **(2 marks)**

b find the three roots of $f(z) = 0$ **(4 marks)**

c find the sum of the three roots of $f(z) = 0$. **(1 mark)**

6 $g(z) = z^3 - 12z^2 + cz + d = 0$, where $c, d \in \mathbb{R}$.

Given that 6 and $3 + i$ are roots of the equation $g(z) = 0$,

a write down the other complex root of the equation **(1 mark)**

b find the value of c and the value of d. **(4 marks)**

7 $h(z) = 2z^3 + 3z^2 + 3z + 1$

Given that $2z + 1$ is a factor of $h(z)$, find the three roots of $h(z) = 0$. **(4 marks)**

8 $f(z) = z^3 - 6z^2 + 28z + k$

Given that $f(2) = 0$,

a find the value of k **(1 mark)**

b find the other two roots of the equation. **(4 marks)**

9 Find the four roots of the equation $z^4 - 16 = 0$.

10 $f(z) = z^4 - 12z^3 + 31z^2 + 108z - 360$

a Write $f(z)$ in the form $(z^2 - 9)(z^2 + bz + c)$, where b and c are real constants to be found. **(2 marks)**

b Hence find all the solutions to $f(z) = 0$. **(3 marks)**

11 $g(z) = z^4 + 2z^3 - z^2 + 38z + 130$

Given that $g(2 + 3i) = 0$, find all the roots of $g(z) = 0$.

12 $f(z) = z^4 - 10z^3 + 71z^2 + Qz + 442$, where Q is a real constant.

Given that $z = 2 - 3i$ is a root of the equation $f(z) = 0$,

a show that $z^2 - 6z + 34$ is a factor of $f(z)$ **(4 marks)**

b find the value of Q **(1 mark)**

c solve completely the equation $f(z) = 0$. **(2 marks)**

ANSWERS

Exercise 1F

1 **a** $f(2) = 8 - 24 + 42 - 26 = 0$

b $z = 2, z = 2 + 3i$ or $z = 2 - 3i$

2 **a** Substitute $z = \frac{1}{2}$ into $f(z)$.

b $b = 3, c = 6$

c $z = \frac{1}{2}$, or $z = -\frac{3}{2} \pm \frac{\sqrt{15}}{2}i$

3 $3, -\frac{1}{2} + \frac{1}{2}i$ and $-\frac{1}{2} - \frac{1}{2}i$

4 **a** $(z - (-4 + i))(z - (-4 - i)) = z^2 + 8z + 16 + 1$

$= z^2 + 8z + 17$

b $z = 4, z = -4 + i$ or $z = -4 - i$

5 **a** $a = 8, b = 25$ **b** $-1, -4 + 3i, -4 - 3i$ **c** -9

6 **a** $3 - i$ **b** $c = 46, d = -60$

7 $-\frac{1}{2}, -\frac{1}{2} + \frac{\sqrt{3}}{2}i$ and $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$

8 **a** $k = -40$ **b** $2 - 4i, 2 + 4i$

9 $2, -2, 2i$ and $-2i$

10 **a** $(z^2 - 9)(z^2 - 12z + 40)$ **b** $z = \pm 3, 6 \pm 2i$

11 $-3 + i, -3 - i, 2 + 3i$ and $2 - 3i$

$(z - (2 - 3i))(z - (2 + 3i)) = z^2 - 4z + 13$

12 **a** $(z^2 - 4z + 13)(z^2 + bz + c)$

$= z^4 - 10z^3 + 71z^2 + Qz + 442$

$b = -6, c = 34$

b $Q = -214$ **c** $z = 2 + 3i, 2 - 3i, 3 + 5i$ or $3 - 5i$

-Edexcel Book Mixed Exercise 1

Mixed exercise 1

1 Given that $z_1 = 8 - 3i$ and $z_2 = -2 + 4i$, find, in the form $a + bi$, where $a, b \in \mathbb{R}$:

a $z_1 + z_2$

b $3z_2$

c $6z_1 - z_2$

2 The equation $z^2 + bz + 14 = 0$, where $b \in \mathbb{R}$ has no real roots.
Find the range of possible values of b. **(3 marks)**

3 The solutions to the quadratic equation $z^2 - 6z + 12 = 0$ are z_1 and z_2.
Find z_1 and z_2, giving each answer in the form $a \pm i\sqrt{b}$.

4 By using the binomial expansion, or otherwise, show that $(1 + 2i)^5 = 41 - 38i$. **(3 marks)**

5 $f(z) = z^2 - 6z + 10$
Show that $z = 3 + i$ is a solution to $f(z) = 0$. **(2 marks)**

6 $z_1 = 4 + 2i$, $z_2 = -3 + i$
Express, in the form $a + bi$, where $a, b \in \mathbb{R}$:

a z_1^* **b** $z_1 z_2$ **c** $\frac{z_1}{z_2}$

7 Write $\frac{(7 - 2i)^2}{1 + i\sqrt{3}}$ in the form $x + iy$ where $x, y \in \mathbb{R}$.

8 Given that $\frac{4 - 7i}{z} = 3 + i$, find z in the form $a + bi$, where $a, b \in \mathbb{R}$. **(2 marks)**

9 $z = \frac{1}{2 + i}$
Express in the form $a + bi$, where $a, b \in \mathbb{R}$:

a z^2 **b** $z - \frac{1}{z}$

10 Given that $z = a + b\text{i}$, show that $\frac{z}{z^*} = \left(\frac{a^2 - b^2}{a^2 + b^2}\right) + \left(\frac{2ab}{a^2 + b^2}\right)\text{i}$ **(4 marks)**

11 The complex number z is defined by $z = \frac{3 + q\text{i}}{q - 5\text{i}}$, where $q \in \mathbb{R}$.

Given that the real part of z is $\frac{1}{13}$,

a find the possible values of q **(4 marks)**

b write the possible values of z in the form $a + b\text{i}$, where a and b are real constants. **(1 mark)**

12 Given that $z = x + \text{i}y$, find the value of x and the value of y such that $z + 4\text{i}z^* = -3 + 18\text{i}$ where z^* is the complex conjugate of z. **(5 marks)**

13 $z = 9 + 6\text{i}$, $w = 2 - 3\text{i}$

Express $\frac{z}{w}$ in the form $a + b\text{i}$, where a and b are real constants.

14 The complex number z is given by $z = \frac{q + 3\text{i}}{4 + q\text{i}}$ where q is an integer.

Express z in the form $a + b\text{i}$ where a and b are rational and are given in terms of q. **(4 marks)**

15 Given that $6 - 2\text{i}$ is one of the roots of a quadratic equation with real coefficients,

a write down the other root of the equation **(1 mark)**

b find the quadratic equation, giving your answer in the form $z^2 + bz + c = 0$ where b and c are real constants. **(2 marks)**

16 Given that $z = 4 - k\text{i}$ is a root of the equation $z^2 - 2mz + 52 = 0$, where k and m are positive real constants, find the value of k and the value of m. **(4 marks)**

17 $\text{h}(z) = z^3 - 11z + 20$

Given that $2 + \text{i}$ is a root of the equation $\text{h}(z) = 0$, solve $\text{h}(z) = 0$ completely. **(4 marks)**

18 $\text{f}(z) = z^3 + 6z + 20$

Given that $\text{f}(1 + 3\text{i}) = 0$, solve $\text{f}(z) = 0$ completely. **(4 marks)**

19 $\text{f}(z) = z^3 + 3z^2 + kz + 48$, $k \in \mathbb{R}$

Given that $\text{f}(4\text{i}) = 0$,

a find the value of k **(2 marks)**

b find the other two roots of the equation. **(3 marks)**

20 $\text{f}(z) = z^4 - z^3 - 16z^2 - 74z - 60$

a Write $\text{f}(z)$ in the form $(z^2 - 5z - 6)(z^2 + bz + c)$, where b and c are real constants to be found. **(2 marks)**

b Hence find all the solutions to $\text{f}(z) = 0$. **(3 marks)**

21 $\text{g}(z) = z^4 - 6z^3 + 19z^2 - 36z + 78$

Given that $\text{g}(3 - 2\text{i}) = 0$, find all the roots of $\text{g}(z) = 0$. **(4 marks)**

22 $\text{f}(z) = z^4 - 2z^3 - 5z^2 + pz + 24$

Given that $\text{f}(4) = 0$,

a find the value of p **(1 mark)**

b solve completely the equation $\text{f}(z) = 0$. **(5 marks)**

ANSWERS

Mixed exercise

1 **a** $6 + i$ **b** $-6 + 12i$ **c** $50 - 22i$

2 $-2\sqrt{14} < b < 2\sqrt{14}$

3 $3 + i\sqrt{3}, 3 - i\sqrt{3}$

4 $(1 + 2i)^5$

$= 1^5 + 5(1)^4(2i) + 10(1)^3(2i)^2 + 10(1)^2(2i)^3 + 5(1)(2i)^4 + (2i)^5$

$= 1 + 10i + 40i^2 + 80i^3 + 80i^4 + 32i^5$

$= 1 + 10i - 40 - 80i + 80 + 32i$

$= 41 - 38i$

5 Substitute $z = 3 + i$ into $f(z)$ to get $f(z) = 0$.

6 **a** $4 - 2i$ **b** $-14 - 2i$ **c** $-1 - i$

7 $\dfrac{(45 - 28i)(1 - i\sqrt{3})}{(1 + \sqrt{3}i)(1 - \sqrt{3}i)} = \dfrac{45 - 28\sqrt{3}}{4} + \left(\dfrac{-45\sqrt{3} - 28}{4}\right)i$

8 $\dfrac{4 - 7i}{3 + i} = \dfrac{(4 - 7i)(3 - i)}{(3 + i)(3 - i)} = \dfrac{12 - 25i + 7i^2}{10} = \frac{1}{2} - \frac{5}{2}i$

9 **a** $\frac{3}{25} - \frac{4}{25}i$ **b** $\frac{-8}{5} - \frac{6}{5}i$

10 $\dfrac{z}{z^*} = \dfrac{(a + bi)(a + bi)}{(a - bi)(a + bi)} = \dfrac{a^2 + 2abi + b^2i^2}{a^2 - b^2i^2}$

$= \dfrac{a^2 - b^2}{a^2 + b^2} + \left(\dfrac{2ab}{a^2 + b^2}\right)i$

11 **a** $\dfrac{3 + qi}{q - 5i} \times \dfrac{q + 5i}{q + 5i} = \dfrac{3q - 5q}{q^2 + 25} + \dfrac{q^2 + 15}{q^2 + 25}i$

$\dfrac{-2q}{q^2 + 25} = \frac{1}{13} \Rightarrow q^2 + 26q + 25 = 0 \Rightarrow q = -1, q = -25$

b $\frac{1}{13} + \frac{8}{13}i, \frac{1}{13} + \frac{64}{65}i$

12 $x + yi + 4i(x - yi) = -3 + 18i$

$(x + 4y) + (4x + y)i = -3 + 18i$

$x + 4y = -3, 4x + y = 18 \Rightarrow x = 5, y = -2$

13 $\dfrac{(9 + 6i)(2 + 3i)}{(2 - 3i)(2 + 3i)} = \dfrac{18 + 39i + 18i^2}{4 - 9i^2} = 3i$

14 $\dfrac{(q + 3i)(4 - qi)}{(4 + qi)(4 - qi)} = \dfrac{7q}{q^2 + 16} + \dfrac{12 - q^2}{q^2 + 16}i$

15 **a** $6 + 2i$ **b** $z^2 - 12z + 40$

16 $k = 6, m = 4$

17 $z = 2 + i, 2 - i$ or -4

18 $z = -2, 1 + 3i$ or $1 - 3i$

19 **a** $k = 16$ **b** $-4i$ and -3

20 **a** $b = 4, c = 10$ **b** $z = 6, -1, -2 + \sqrt{6}i$ or $-2 - \sqrt{6}i$

21 $3 - 2i, 3 + 2i, i\sqrt{6}$ and $-i\sqrt{6}$

22 **a** $p = -18$ **b** $1, 4, -\frac{3}{2} + \dfrac{\sqrt{15}}{2}i$ and $-\frac{3}{2} - \dfrac{\sqrt{15}}{2}i$

-Edexcel Book Ex2A Q2-6

using GeoGebra.

Exercise 2A

1 Show these numbers on an Argand diagram.

a $7 + 2i$ b $5 - 4i$ c $-6 - i$ d $-2 + 5i$

e $3i$ f $\sqrt{2} + 2i$ g $-\frac{1}{2} + \frac{5}{2}i$ h -4

2 $z_1 = 11 + 2i$ and $z_2 = 2 + 4i$. Show z_1, z_2 and $z_1 + z_2$ on an Argand diagram.

3 $z_1 = -3 + 6i$ and $z_2 = 8 - i$. Show z_1, z_2 and $z_1 + z_2$ on an Argand diagram.

4 $z_1 = 8 + 4i$ and $z_2 = 6 + 7i$. Show z_1, z_2 and $z_1 - z_2$ on an Argand diagram.

5 $z_1 = -6 - 5i$ and $z_2 = -4 + 4i$. Show z_1, z_2 and $z_1 - z_2$ on an Argand diagram.

6 $z_1 = 7 - 5i$, $z_2 = a + bi$ and $z_3 = -3 + 2i$ where $a, b \in \mathbb{Z}$. Given that $z_3 = z_1 + z_2$,

a find the values of a and b b show z_1, z_2 and z_3 on an Argand diagram.

7 $z_1 = p + qi$, $z_2 = 9 - 5i$ and $z_3 = -8 + 5i$ where $p, q \in \mathbb{Z}$. Given that $z_3 = z_1 + z_2$,

a find the values of p and q b show z_1, z_2 and z_3 on an Argand diagram.

8 The solutions to the quadratic equation $z^2 - 6z + 10 = 0$ are z_1 and z_2.

a Find z_1 and z_2, giving your answers in the form $p \pm qi$, where p and q are integers. **(3 marks)**

b Show, on an Argand diagram, the points representing the complex numbers z_1 and z_2. **(2 marks)**

9 $f(z) = 2z^3 - 19z^2 + 64z - 60$

a Show that $f\left(\frac{3}{2}\right) = 0$. **(1 mark)**

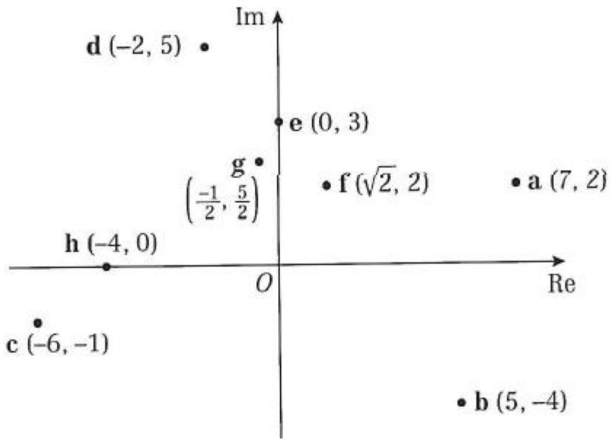
b Use algebra to solve $f(z) = 0$ completely. **(4 marks)**

c Show all three solutions on an Argand diagram. **(2 marks)**

ANSWERS

Exercise 2A

1



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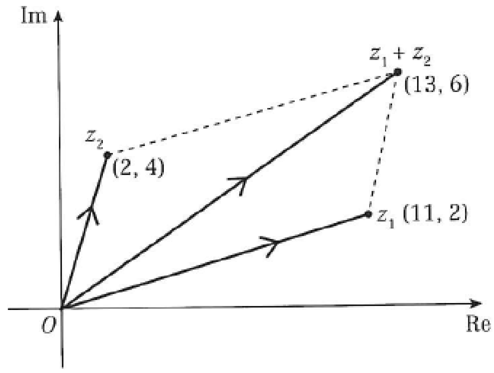

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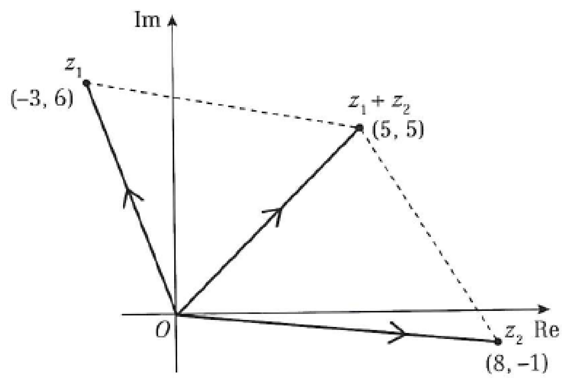

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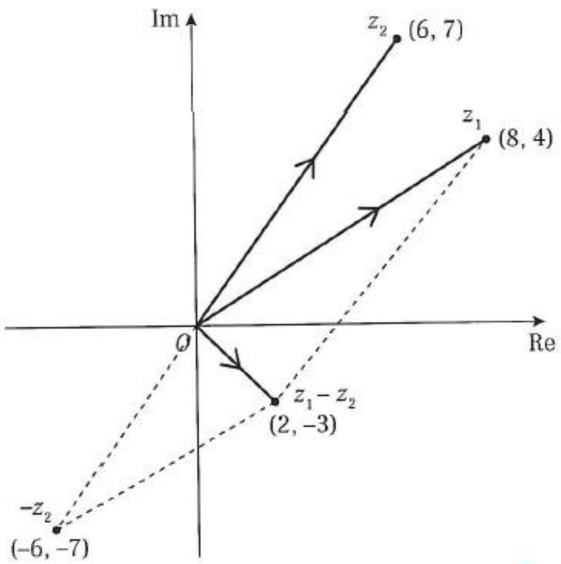

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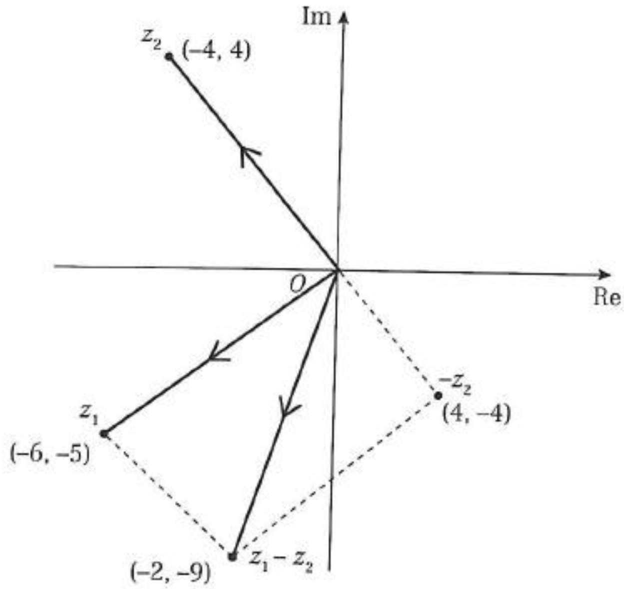
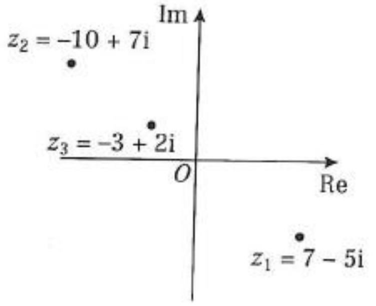


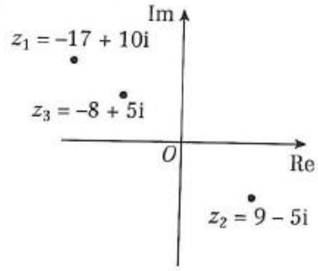
6 **a** $a = -10$, $b = 7$

b



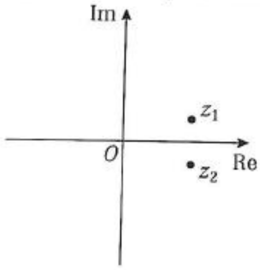
7 **a** $p = -17$, $q = 10$

b



8 **a** $z_1 = 3 + i$ and $z_2 = 3 - i$. Other way round acceptable.

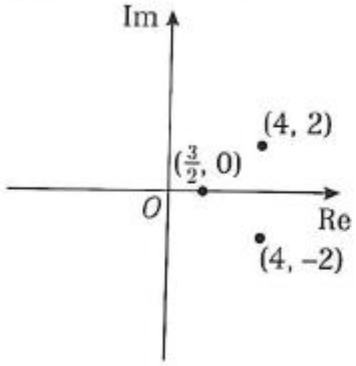
b



9 **a** $2\left(\frac{3}{2}\right)^3 - 19\left(\frac{3}{2}\right)^2 + 64\left(\frac{3}{2}\right) - 60 = 0$

b $\left(\frac{3}{2}\right)$, $4 + 2i$, $4 - 2i$.

c



-Edexcel Book Ex2C Q1-3(alt parts), Q4-6

Exercise 2C

1 Express the following in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $-\pi < \theta \leqslant \pi$. Give the exact values of r and θ where possible, or values to 2 decimal places otherwise.

a $2 + 2\mathrm{i}$ **b** $3\mathrm{i}$ **c** $-3 + 4\mathrm{i}$ **d** $1 - \sqrt{3}\mathrm{i}$

e $-2 - 5\mathrm{i}$ **f** -20 **g** $7 - 24\mathrm{i}$ **h** $-5 + 5\mathrm{i}$

2 Express these in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, giving exact values of r and θ where possible, or values to two decimal places otherwise.

a $\dfrac{3}{1 + \mathrm{i}\sqrt{3}}$ **b** $\dfrac{1}{2 - \mathrm{i}}$ **c** $\dfrac{1 + \mathrm{i}}{1 - \mathrm{i}}$

3 Express the following in the form $x + \mathrm{i}y$, where $x, y \in \mathbb{R}$.

a $5\left(\cos\frac{\pi}{2} + \mathrm{i}\sin\frac{\pi}{2}\right)$ **b** $\frac{1}{2}\left(\cos\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{6}\right)$ **c** $6\left(\cos\frac{5\pi}{6} + \mathrm{i}\sin\frac{5\pi}{6}\right)$

d $3\left(\cos\left(-\frac{2\pi}{3}\right) + \mathrm{i}\sin\left(-\frac{2\pi}{3}\right)\right)$ **e** $2\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + \mathrm{i}\sin\left(-\frac{\pi}{4}\right)\right)$ **f** $-4\left(\cos\frac{7\pi}{6} + \mathrm{i}\sin\frac{7\pi}{6}\right)$

4 **a** Express the complex number $z = 4\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)$ in the form $x + \mathrm{i}y$, where $x, y \in \mathbb{R}$. **(2 marks)**

b Show the complex number z on an Argand diagram. **(1 mark)**

5 The complex number z is such that $|z| = 7$ and $\arg z = \frac{11\pi}{6}$. Find z in the form $p + q\mathrm{i}$, where p and q are exact real numbers to be found. **(3 marks)**

6 The complex number z is such that $|z| = 5$ and $\arg z = -\frac{4\pi}{3}$. Find z in the form $a + b\mathrm{i}$, where a and b are exact real numbers to be found. **(3 marks)**

ANSWERS

Exercise 2C

1 **a** $2\sqrt{2}\left(\cos\frac{\pi}{4} + \mathrm{i}\sin\frac{\pi}{4}\right)$ **b** $3\left(\cos\frac{\pi}{2} + \mathrm{i}\sin\frac{\pi}{2}\right)$

c $5(\cos 2.21 + \mathrm{i}\sin 2.21)$ **d** $2\left(\cos -\frac{\pi}{3} + \mathrm{i}\sin -\frac{\pi}{3}\right)$

e $\sqrt{29}(\cos(-1.95) + \mathrm{i}\sin(-1.95))$

f $20(\cos\pi + \mathrm{i}\sin\pi)$

g $25(\cos(-1.29) + \mathrm{i}\sin(-1.29))$

h $5\sqrt{2}\left(\cos\frac{3\pi}{4} + \mathrm{i}\sin\frac{3\pi}{4}\right)$

2 **a** $\frac{3}{2}\left(\cos\left(-\frac{\pi}{3}\right) + \mathrm{i}\sin\left(-\frac{\pi}{3}\right)\right)$ **b** $\frac{\sqrt{5}}{5}(\cos 0.46 + \mathrm{i}\sin 0.46)$

c $1\left(\cos\frac{\pi}{2} + \mathrm{i}\sin\frac{\pi}{2}\right)$

3 **a** $5\mathrm{i}$ **b** $\frac{\sqrt{3}}{4} + \frac{1}{4}\mathrm{i}$ **c** $-3\sqrt{3} + 3\mathrm{i}$

d $-\frac{3}{2} - \frac{3\sqrt{3}}{2}\mathrm{i}$ **e** $2 - 2\mathrm{i}$ **f** $2\sqrt{3} + 2\mathrm{i}$

4 **a** $-2 + 2\mathrm{i}\sqrt{3}$

b $-2 + 2\mathrm{i}\sqrt{3}$ shown on an Argand diagram.

5 $p = \frac{7\sqrt{3}}{2}$, $q = -\frac{7}{2}$

6 $a = -\frac{5}{2}$, $b = \frac{5\sqrt{3}}{2}$

-Edexcel Book Ex2D Q1a,23ace,4ac

Exercise 2D

1 For each given z_1 and z_2, find the following in the form $r(\cos\theta + \mathrm{i}\sin\theta)$:

i $|z_1 z_2|$ **ii** $\arg(z_1 z_2)$ **iii** $z_1 z_2$

a $z_1 = 5\left(\cos\frac{3\pi}{8} + \mathrm{i}\sin\frac{3\pi}{8}\right)$, $z_2 = 6\left(\cos\frac{7\pi}{8} + \mathrm{i}\sin\frac{7\pi}{8}\right)$

b $z_1 = \sqrt{2}\left(\cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3}\right)$, $z_2 = 4\sqrt{2}\left(\cos\frac{3\pi}{4} + \mathrm{i}\sin\frac{3\pi}{4}\right)$

2 Given $z_1 = 8\left(\cos\frac{8\pi}{5} + \mathrm{i}\sin\frac{8\pi}{5}\right)$ and $z_2 = 4\left(\cos\frac{2\pi}{3} + \mathrm{i}\sin\frac{2\pi}{3}\right)$, write down the modulus and argument of:

a $z_1 z_2$ **b** $\frac{z_1}{z_2}$ **c** z_1^2

3 Express the following in the form $x + \mathrm{i}y$:

a $(\cos 2\theta + \mathrm{i}\sin 2\theta)(\cos 3\theta + \mathrm{i}\sin 3\theta)$

b $\left(\cos\frac{3\pi}{11} + \mathrm{i}\sin\frac{3\pi}{11}\right)\left(\cos\frac{8\pi}{11} + \mathrm{i}\sin\frac{8\pi}{11}\right)$

c $3\left(\cos\frac{\pi}{4} + \mathrm{i}\sin\frac{\pi}{4}\right) \times 2\left(\cos\frac{\pi}{12} + \mathrm{i}\sin\frac{\pi}{12}\right)$

d $\sqrt{6}\left(\cos\frac{\pi}{3} - \mathrm{i}\sin\frac{\pi}{3}\right) \times \sqrt{3}\left(\cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3}\right)$

e $4\left(\cos\frac{5\pi}{9} - \mathrm{i}\sin\frac{5\pi}{9}\right) \times \frac{1}{2}\left(\cos\frac{5\pi}{18} - \mathrm{i}\sin\frac{5\pi}{18}\right)$

f $6\left(\cos\frac{\pi}{10} + \mathrm{i}\sin\frac{\pi}{10}\right) \times 5\left(\cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3}\right) \times \frac{1}{3}\left(\cos\frac{2\pi}{5} + \mathrm{i}\sin\frac{2\pi}{5}\right)$

Hint First make sure both numbers are in modulus–argument form.

g $(\cos 4\theta + \mathrm{i}\sin 4\theta)(\cos\theta - \mathrm{i}\sin\theta)$

h $3\left(\cos\frac{\pi}{12} + \mathrm{i}\sin\frac{\pi}{12}\right) \times \sqrt{2}\left(\cos\frac{\pi}{3} - \mathrm{i}\sin\frac{\pi}{3}\right)$

4 Express the following in the form $x + \mathrm{i}y$:

a $\dfrac{\cos 5\theta + \mathrm{i}\sin 5\theta}{\cos 2\theta + \mathrm{i}\sin 2\theta}$ **b** $\dfrac{\sqrt{2}\left(\cos\frac{\pi}{2} + \mathrm{i}\sin\frac{\pi}{2}\right)}{\frac{1}{2}\left(\cos\frac{\pi}{4} + \mathrm{i}\sin\frac{\pi}{4}\right)}$ **c** $\dfrac{3\left(\cos\frac{\pi}{3} + \mathrm{i}\sin\frac{\pi}{3}\right)}{4\left(\cos\frac{5\pi}{6} + \mathrm{i}\sin\frac{5\pi}{6}\right)}$ **d** $\dfrac{\cos 2\theta - \mathrm{i}\sin 2\theta}{\cos 3\theta + \mathrm{i}\sin 3\theta}$

5 $z = -9 + 3\mathrm{i}\sqrt{3}$

a Express z in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, $-\pi < \theta \leqslant \pi$ **(2 marks)**

b Given that $|w| = \sqrt{3}$ and $\arg w = \frac{7\pi}{12}$, express in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, $-\pi < \theta \leqslant \pi$:

i w **ii** zw **iii** $\frac{z}{w}$ **(4 marks)**

ANSWERS

Exercise 2D

1 **a** i $|z_1 z_2| = 30$ ii $\arg(z_1 z_2) = \frac{5\pi}{4}$
iii $30\left(\cos\frac{5\pi}{4} + \mathrm{i}\sin\frac{5\pi}{4}\right)$
b i $|z_1 z_2| = 8$ ii $\arg(z_1 z_2) = \frac{13\pi}{12}$
iii $8\left(\cos\frac{13\pi}{12} + \mathrm{i}\sin\frac{13\pi}{12}\right)$

2 **a** $|z_1 z_2| = 32$, $\arg(z_1 z_2) = \frac{4\pi}{15}$
b $\left|\frac{z_1}{z_2}\right| = 2$, $\arg\left(\frac{z_1}{z_2}\right) = \frac{14\pi}{15}$
c $|z_1^2| = 64$, $\arg(z_1^2) = -\frac{4\pi}{5}$

3 **a** $\cos 5\theta + \mathrm{i}\sin 5\theta$ **b** -1 **c** $-\frac{3}{4}\mathrm{i}$
d $3\sqrt{2}$ **e** $-\sqrt{3} - \mathrm{i}$ **f** $-5\sqrt{3} + 5\mathrm{i}$
g $\cos 3\theta + \mathrm{i}\sin 3\theta$ **h** $3 - 3\mathrm{i}$

4 **a** $\cos 3\theta + \mathrm{i}\sin 3\theta$ **b** $2 + 2\mathrm{i}$ **c** $-\frac{11}{4}\mathrm{i}$
d $\cos(-5\theta) + \mathrm{i}\sin(-5\theta)$ or $\cos 5\theta - \mathrm{i}\sin 5\theta$

5 **a** $z = 6\sqrt{3}\left(\cos\frac{5\pi}{6} + \mathrm{i}\sin\frac{5\pi}{6}\right)$
b i $w = \sqrt{3}\left(\cos\frac{7\pi}{12} + \mathrm{i}\sin\frac{7\pi}{12}\right)$
ii $18\cos\left(-\frac{7\pi}{12}\right) + \mathrm{i}\sin\left(-\frac{7\pi}{12}\right)$
iii $6\left(\cos\frac{\pi}{4} + \mathrm{i}\sin\frac{\pi}{4}\right)$

-Edexcel Book Ex2E Q1acei,4,6acei,7,10ace

Exercise 2E

1 Sketch the locus of z and give the Cartesian equation of the locus of z when:

a $|z| = 6$　　b $|z| = 10$　　c $|z - 3| = 2$

d $|z + 3i| = 3$　　e $|z - 4i| = 5$　　f $|z + 1| = 1$

g $|z - 1 - i| = 5$　　h $|z + 3 + 4i| = 4$　　i $|z - 5 + 6i| = 5$

Hint You may choose a geometric or an algebraic approach to answer these questions.

2 Given that z satisfies $|z - 5 - 4i| = 8$,

a sketch the locus of z on an Argand diagram

b find the exact values of z that satisfy:

i both $|z - 5 - 4i| = 8$ and $\text{Re}(z) = 0$　　ii both $|z - 5 - 4i| = 8$ and $\text{Im}(z) = 0$

3 A complex number z is represented by the point P on the Argand diagram.
Given that $|z - 5 + 7i| = 5$,

a sketch the locus of P

b find the Cartesian equation of this locus

c find the maximum value of $\arg z$ in the interval $(-\pi, \pi)$.

4 On an Argand diagram the point P represents the complex number z.
Given that $|z - 4 - 3i| = 8$,

a find the Cartesian equation for the locus of P **(2 marks)**

b sketch the locus of P **(2 marks)**

c find the maximum and minimum values of $|z|$ for points on this locus. **(2 marks)**

5 The point P represents a complex number z on an Argand diagram.
Given that $|z + 2 - 2\sqrt{3}i| = 2$,

a sketch the locus of P on an Argand diagram **(2 marks)**

b write down the minimum value of $\arg z$ **(2 marks)**

c find the maximum value of $\arg z$. **(2 marks)**

6 Sketch the locus of z and give the Cartesian equation of the locus of z when:

a $|z - 6| = |z - 2|$　　b $|z + 8| = |z - 4|$

c $|z| = |z + 6i|$　　d $|z + 3i| = |z - 8i|$

e $|z - 2 - 2i| = |z + 2 + 2i|$　　f $|z + 4 + i| = |z + 4 + 6i|$

g $|z + 3 - 5i| = |z - 7 - 5i|$　　h $|z + 4 - 2i| = |z - 8 + 2i|$

i $\dfrac{|z + 3|}{|z - 6i|} = 1$　　j $\dfrac{|z + 6 - i|}{|z - 10 - 5i|} = 1$

7 Given that $|z-3| = |z-6i|$,

a sketch the locus of z **(3 marks)**

b find the exact least possible value of $|z|$. **(4 marks)**

8 Given that $|z+3+3i| = |z-9-5i|$,

a sketch the locus of z **(3 marks)**

b find the Cartesian equation of this locus **(3 marks)**

c find the exact least possible value of $|z|$. **(3 marks)**

9 Sketch the locus of z and give the Cartesian equation of the locus of z when:

a $|2-z| = 3$ b $|5i-z| = 4$ c $|3-2i-z| = 3$

10 Sketch the locus of z when:

a $\arg z = \frac{\pi}{3}$ b $\arg(z+3) = \frac{\pi}{4}$ c $\arg(z-2) = \frac{\pi}{2}$

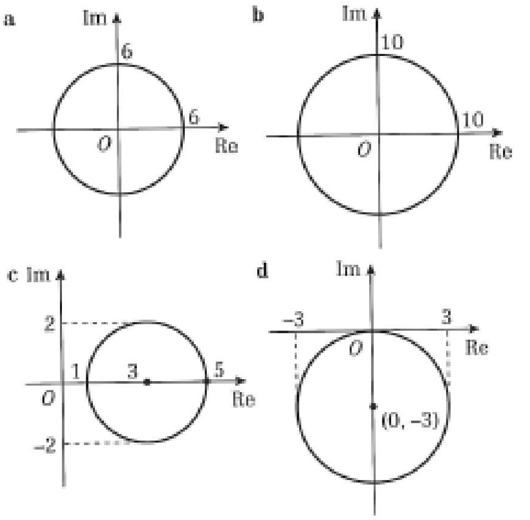
d $\arg(z+2+2i) = -\frac{\pi}{4}$ e $\arg(z-1-i) = \frac{3\pi}{4}$ f $\arg(z+3i) = \pi$

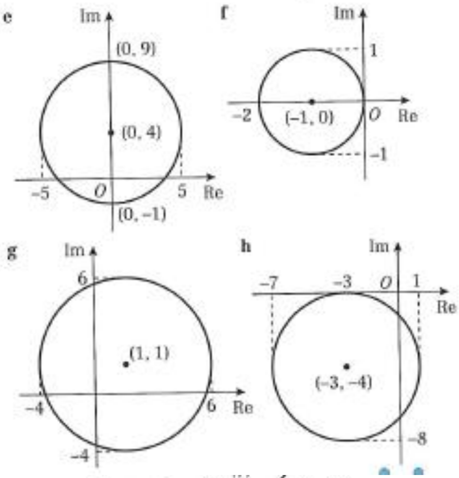
g $\arg(z-1+3i) = \frac{2\pi}{3}$ h $\arg(z-3+4i) = -\frac{\pi}{2}$ i $\arg(z-4i) = -\frac{3\pi}{4}$

<u>ANSWERS</u>

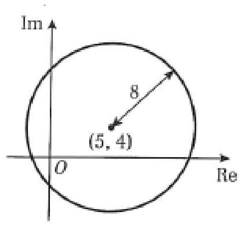
Exercise 2E

1



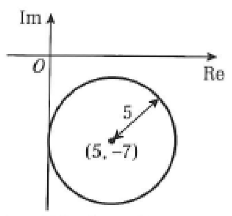


2 a



b i $(4+\sqrt{39})i$ and $(4-\sqrt{39})i$
ii $5+4\sqrt{3}$ and $5-4\sqrt{3}$

3 a

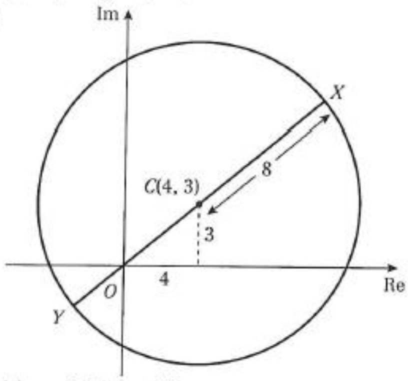


b $(x-5)^2 + (y+7)^2 = 25$

c $2\arctan\left(\frac{5}{7}\right) - \frac{\pi}{2} = -0.330$ rad (3 s.f.)

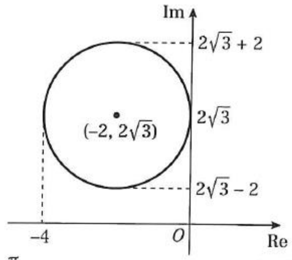
4 a $(x-4)^2 + (y-3)^2 = 8^2$

b



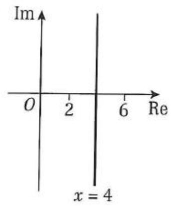
c $|z|_{min} = 3$, $|z|_{max} = 13$

5 **a**



b $\frac{\pi}{2}$ **c** 2.51 rad

6 **a**



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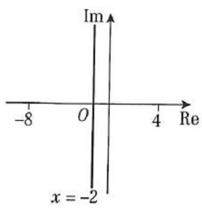

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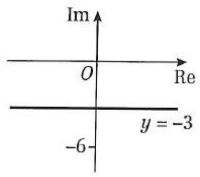

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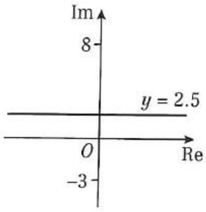

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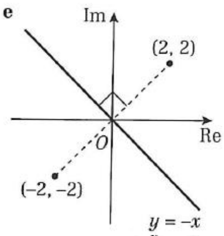

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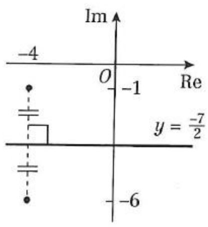

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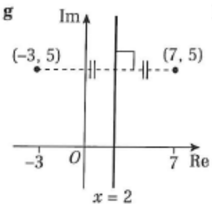

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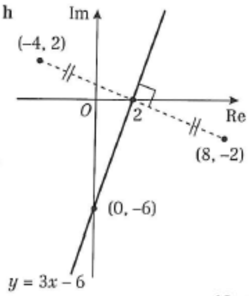

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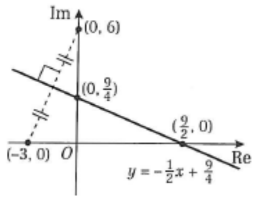

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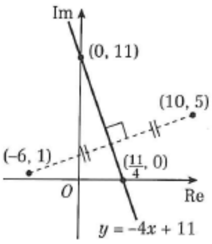
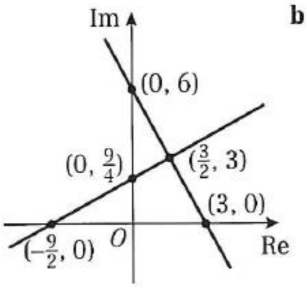
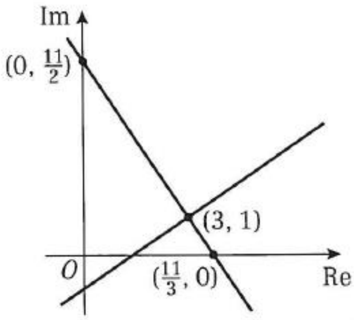


7 **a**



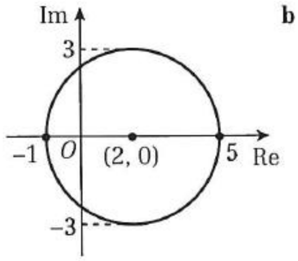
b $\frac{9\sqrt{5}}{10}$

8 **a**



b $y = -\frac{3}{2}x + \frac{11}{2}$ **c** $\frac{11\sqrt{13}}{13}$

9 **a**



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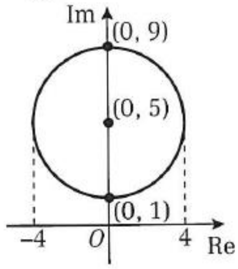

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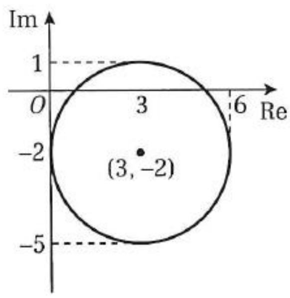
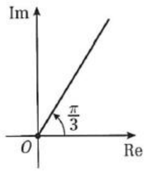


10 **a**



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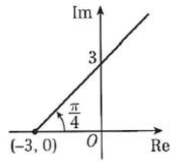

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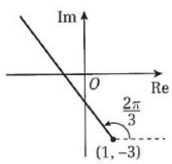

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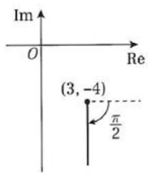

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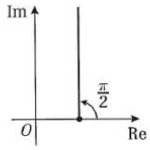

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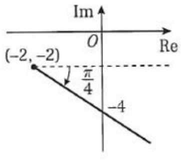

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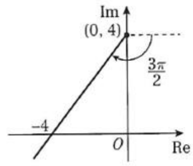

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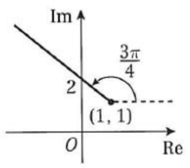

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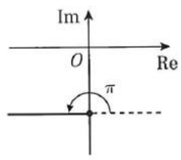


-Edexcel Book Ex2F Q1aceg,2-6

Exercise 2F

1 On an Argand diagram, shade in the regions represented by the following inequalities:

a $|z| < 3$ **b** $|z - 2\mathrm{i}| > 2$ **c** $|z + 7| \geqslant |z - 1|$ **d** $|z + 6| > |z + 2 + 8\mathrm{i}|$

e $2 \leqslant |z| \leqslant 3$ **f** $1 \leqslant |z + 4\mathrm{i}| \leqslant 4$ **g** $3 \leqslant |z - 3 + 5\mathrm{i}| \leqslant 5$

2 The region R in an Argand diagram is satisfied by the inequalities $|z| \leqslant 5$ and $|z| \leqslant |z - 6\mathrm{i}|$.
Draw an Argand diagram and shade in the region R. **(6 marks)**

3 The complex number z is represented by a point P on an Argand diagram.
Given that $|z + 1 - \mathrm{i}| \leqslant 1$ and $0 \leqslant \arg z \leqslant \frac{3\pi}{4}$, shade the locus of P. **(6 marks)**

4 Shade on an Argand diagram the region satisfied by

$$\{z \in \mathbb{C} : |z| \leqslant 3\} \cap \left\{z \in \mathbb{C} : \frac{\pi}{4} \leqslant \arg(z + 3) \leqslant \pi\right\}$$ **(6 marks)**

5 **a** Sketch on the same Argand diagram:

i the locus of points representing $|z - 2| = |z - 6 - 8\mathrm{i}|$ **(2 marks)**

ii the locus of points representing $\arg(z - 4 - 2\mathrm{i}) = 0$ **(2 marks)**

iii the locus of points representing $\arg(z - 4 - 2\mathrm{i}) = \frac{\pi}{2}$ **(2 marks)**

b Shade on an Argand diagram the set of points

$$\{z \in \mathbb{C} : |z - 2| \leqslant |z - 6 - 8\mathrm{i}|\} \cap \left\{z \in \mathbb{C} : 0 \leqslant \arg(z - 4 - 2\mathrm{i}) \leqslant \frac{\pi}{2}\right\}$$ **(2 marks)**

6 **a** Find the Cartesian equations of:

i the locus of points representing $|z + 10| = |z - 6 - 4\mathrm{i}\sqrt{2}|$

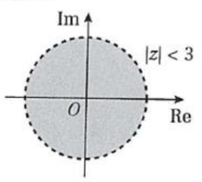
ii the locus of points representing $|z + 1| = 3$. **(6 marks)**

b Find the two values of z that satisfy both $|z + 10| = |z - 6 - 4\mathrm{i}\sqrt{2}|$ and $|z + 1| = 3$. **(2 marks)**

c Hence shade in the region R on an Argand diagram which satisfies both $|z + 10| \leqslant |z - 6 - 4\mathrm{i}\sqrt{2}|$ and $|z + 1| \leqslant 3$. **(4 marks)**

ANSWERS

Exercise 2F

1 a


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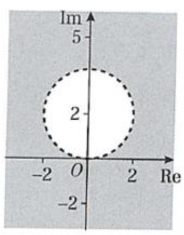

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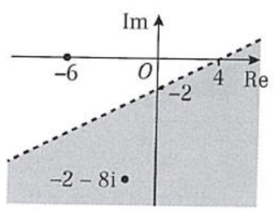

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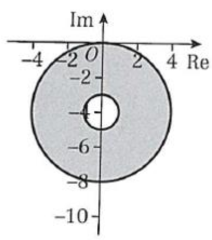

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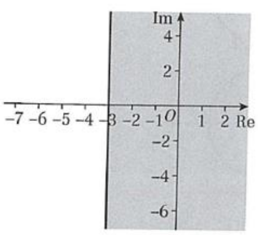

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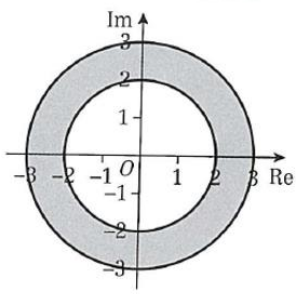
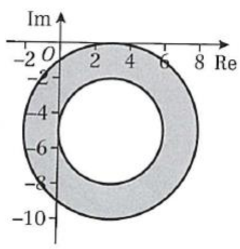


g


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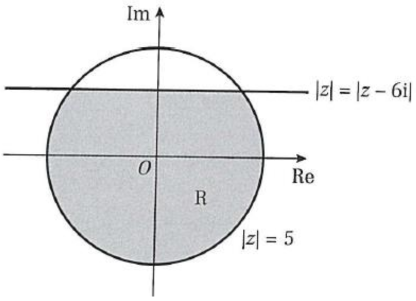

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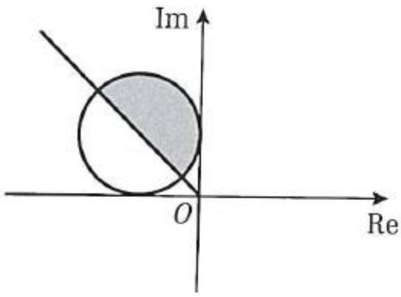

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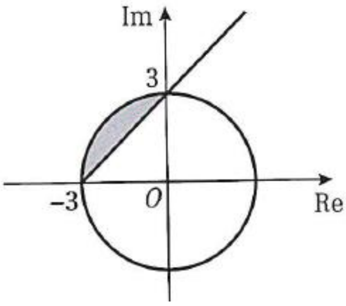

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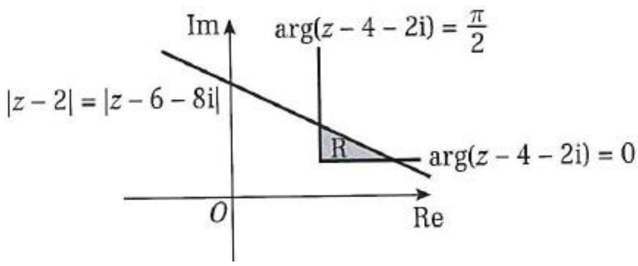
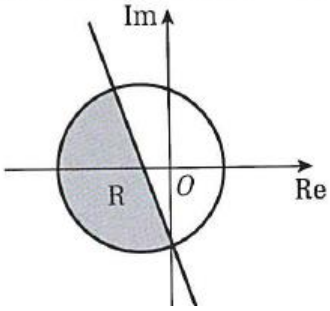


6 **a** **i** $y = -2\sqrt{2}x - 2\sqrt{2}$ **ii** $(x + 1)^2 + y^2 = 9$

b $z = -\sqrt{2} + 2i\sqrt{2}$ or $z = -2i$

c



-Edexcel Book Mixed Exercise 2

Mixed exercise 2

1 $f(z) = z^2 + 5z + 10$

a Find the roots of the equation $f(z) = 0$, giving your answers in the form $a \pm ib$, where a and b are real numbers. **(3 marks)**

b Show these roots on an Argand diagram. **(1 mark)**

2 $f(z) = z^3 + z^2 + 3z - 5$

Given that $f(-1 + 2i) = 0$,

a find all the solutions to the equation $f(z) = 0$ **(4 marks)**

b show all the roots of $f(z) = 0$ on a single Argand diagram **(2 marks)**

c prove that these three points are the vertices of a right-angled triangle. **(2 marks)**

3 $f(z) = z^4 - z^3 + 13z^2 - 47z + 34$

Given that $z = -1 + 4i$ is a solution to the equation,

a find all the solutions to the equation $f(z) = 0$ **(4 marks)**

b show all the roots on a single Argand diagram. **(2 marks)**

4 The real and imaginary parts of the complex number $z = x + iy$ satisfy the equation

$$(4 - 3i)x - (1 + 6i)y - 3 = 0$$

a Find the value of x and the value of y. **(3 marks)**

b Show z on an Argand diagram. **(1 mark)**

Find the values of:

c $|z|$ **(2 marks)**

d $\arg z$ **(2 marks)**

5 $z_1 = 4 + 2i$, $z_2 = -3 + i$

a Draw points representing z_1 and z_2 on the same Argand diagram. **(1 mark)**

b Find the exact value of $|z_1 - z_2|$. **(2 marks)**

Given that $w = \frac{z_1}{z_2}$,

c express w in the form $a + ib$, where $a, b \in \mathbb{R}$ **(2 marks)**

d find $\arg w$, giving your answer in radians. **(2 marks)**

6 A complex number z is given by $z = a + 4i$ where a is a non-zero real number.

a Find $z^2 + 2z$ in the form $x + iy$, where x and y are real expressions in terms of a. **(4 marks)**

Given that $z^2 + 2z$ is real,

b find the value of a. **(1 mark)**

Using this value for a,

c find the values of the modulus and argument of z, giving the argument in radians and giving your answers correct to 3 significant figures. **(3 marks)**

d Show the complex numbers z, z^2 and $z^2 + 2z$ on a single Argand diagram. **(3 marks)**

7 The complex number z is defined by $z = \frac{3 + 5i}{2 - i}$

Find:

a $|z|$ **(4 marks)**

b $\arg z$ **(2 marks)**

8 $z = 1 + 2i$

a Show that $|z^2 - z| = 2\sqrt{5}$. **(4 marks)**

b Find $\arg(z^2 - z)$, giving your answer in radians to 2 decimal places. **(2 marks)**

c Show z and $z^2 - z$ on a single Argand diagram. **(2 marks)**

9 $z = \frac{1}{2 + i}$

a Express in the form $a + bi$, where $a, b \in \mathbb{R}$,

i z^2 **ii** $z - \frac{1}{z}$ **(4 marks)**

b Find $|z^2|$. **(2 marks)**

c Find $\arg\left(z - \frac{1}{z}\right)$, giving your answer in radians to two decimal places. **(2 marks)**

10 $z = \frac{a + 3i}{2 + ai}$, $a \in \mathbb{R}$

a Given that $a = 4$, find $|z|$.

b Show that there is only one value of a for which $\arg z = \frac{\pi}{4}$, and find this value.

11 $z_1 = -1 - \text{i}$, $z_2 = 1 + \text{i}\sqrt{3}$

a Express z_1 and z_2 in the form $r(\cos\theta + \text{i}\sin\theta)$, where $-\pi < \theta \leqslant \pi$. **(2 marks)**

b Find the modulus of:

i z_1z_2 **ii** $\frac{z_1}{z_2}$ **(2 marks)**

c Find the argument of:

i z_1z_2 **ii** $\frac{z_1}{z_2}$ **(2 marks)**

12 $z = 2 - 2\text{i}\sqrt{3}$

Find:

a $|z|$ **(1 mark)**

b $\arg z$, in terms of π. **(2 marks)**

$w = 4\left(\cos\left(-\frac{\pi}{4}\right) + \text{i}\sin\left(-\frac{\pi}{4}\right)\right)$

Find:

c $\left|\frac{w}{z}\right|$ **(1 mark)**

d $\arg\left(\frac{w}{z}\right)$, in terms of π. **(2 marks)**

13 Express $4 - 4\text{i}$ in the form $r(\cos\theta + \text{i}\sin\theta)$, where $r > 0$, $-\pi < \theta \leqslant \pi$, giving r and θ as exact values. **(3 marks)**

14 The point P represents a complex number z in an Argand diagram.
Given that $|z + 1 - \text{i}| = 1$,

a find a Cartesian equation for the locus of P **(2 marks)**

b sketch the locus of P on an Argand diagram **(2 marks)**

c find the greatest and least possible values of $|z|$ **(2 marks)**

d find the greatest and least possible values of $|z - 1|$. **(2 marks)**

15 Given that $\arg(z - 2 + 4\text{i}) = \frac{\pi}{4}$,

a sketch the locus of $P(x, y)$ which represents z on an Argand diagram

b find the minimum value of $|z|$ for points on this locus.

16 The complex number z satisfies $|z + 3 - 6\text{i}| = 3$. Show that the exact maximum value of $\arg z$ in the interval $(-\pi, \pi)$ is $\frac{\pi}{2} + 2\arcsin\left(\frac{1}{\sqrt{5}}\right)$. **(4 marks)**

17 A complex number z is represented by the point P on the Argand diagram.
Given that $|z - 5| = 4$,

a sketch the locus of P. **(2 marks)**

b Find the complex numbers that satisfy both $|z - 5| = 4$ and $\arg(z + 3\text{i}) = \frac{\pi}{3}$, giving your answers in radians to 2 decimal places. **(6 marks)**

c Given that $\arg(z + 5) = \theta$ and $|z - 5| = 4$ have no common solutions, find the range of possible values of θ, $-\pi < \theta < \pi$. **(3 marks)**

18 Given that $|z + 5 - 5\mathrm{i}| = |z - 6 - 3\mathrm{i}|$,

a sketch the locus of z **(3 marks)**

b find the Cartesian equation of this locus **(3 marks)**

c find the least possible value of $|z|$. **(3 marks)**

19 **a** Find the Cartesian equation of the locus of points that satisfies $|z - 4| = |z - 8\mathrm{i}|$. **(3 marks)**

b Find the value of z that satisfies both $|z - 2| = |z - 4\mathrm{i}|$ and $\arg z = \frac{\pi}{4}$ **(3 marks)**

c Shade on an Argand diagram the set of points

$\{z \in \mathbb{C} : |z - 4| \leqslant |z - 8\mathrm{i}|\} \cap \left\{z \in \mathbb{C} : \frac{\pi}{4} \leqslant \arg z \leqslant \pi\right\}$ **(3 marks)**

20 **a** Find the Cartesian equations of:

i the locus of points representing $|z - 3 + \mathrm{i}| = |z - 1 - \mathrm{i}|$

ii the locus of points representing $|z - 2| = 2\sqrt{2}$. **(6 marks)**

b Find the two values of z that satisfy both $|z - 3 + \mathrm{i}| = |z - 1 - \mathrm{i}|$ and $|z - 2| = 2\sqrt{2}$. **(2 marks)**

The region R is defined by the inequalities $|z - 3 + \mathrm{i}| \geqslant |z - 1 - \mathrm{i}|$ and $|z - 2| \leqslant 2\sqrt{2}$.

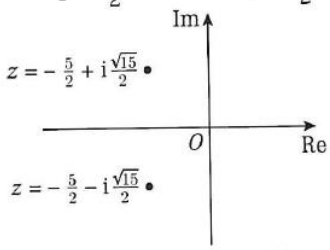
c Show the region R on an Argand diagram. **(4 marks)**

<u>ANSWERS</u>

Mixed exercise 2

1 **a** $z = -\frac{5}{2} + \frac{\sqrt{15}}{2}\mathrm{i}$ and $z = -\frac{5}{2} - \frac{\sqrt{15}}{2}\mathrm{i}$

b



2 **a** $-1 + 2\mathrm{i}, -1 - 2\mathrm{i}$ are two of the roots. These roots can be used to form the quadratic $z^2 + 2z + 5$.
$(z - 1)(z^2 + 2z + 5) = \mathrm{f}(z)$, so third root is 1.

b Argand diagram showing $-1 + 2\mathrm{i}, -1 - 2\mathrm{i}$ and 1.

c Sides of triangle are $\sqrt{8}, \sqrt{8}$ and 4. $(\sqrt{8})^2 + (\sqrt{8})^2 = 4^2$.

3 **a** $-1 + 4\mathrm{i}, -1 - 4\mathrm{i}, 2, 1$

b Argand diagram showing above roots.

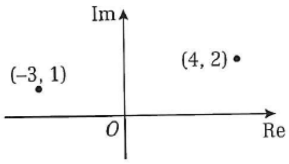
4 **a** $4x - y = 3$
$-3x - 6y = 0 \Rightarrow x = -2y$
$-9y = 3 \Rightarrow y = -\frac{1}{3} \Rightarrow x = \frac{2}{3}$

b Argand diagram showing the point $z = \frac{2}{3} - \frac{1}{3}i$

c $\frac{\sqrt{5}}{3}$

d -0.46 rad

5 **a**



b $5\sqrt{2}$ **c** $-1 - \mathrm{i}$ **d** $-\frac{3\pi}{4}$

6 **a** $z^2 = (a^2 - 16) + 8a\mathrm{i}$
$2z = 2a + 8\mathrm{i}$
$z^2 + 2z = (a^2 + 2a - 16) + (8 + 8a)\mathrm{i}$

b $a = -1$

c $z = -1 + 4\mathrm{i}$
$|z| = \sqrt{17} \approx 4.12$
$\arg z \approx 1.82$

d Show $z = -1 + 4\mathrm{i}$, $z^2 = -15 - 8\mathrm{i}$ and $z^2 + 2z = -17$ on a single Argand diagram.

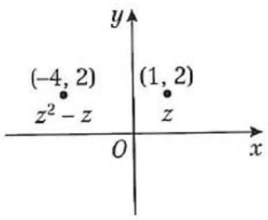
7 **a** $z = \frac{(3 + 5\mathrm{i})(2 + \mathrm{i})}{(2 - \mathrm{i})(2 + \mathrm{i})} = \frac{1}{5} + \frac{13}{5}\mathrm{i}$
$|z| = \frac{1}{5}\sqrt{170}$

b $\arg z = 1.49$

8 **a** $z^2 = -3 + 4\mathrm{i}$
$z^2 - z = -4 + 2\mathrm{i}$
$|-4 + 2\mathrm{i}| = \sqrt{(-4)^2 + (2)^2} = \sqrt{20} = 2\sqrt{5}$

b 2.68

c



9 **a** **i** $\frac{3}{25} - \frac{4}{25}\mathrm{i}$ **ii** $\frac{-8}{5} - \frac{6}{5}\mathrm{i}$ **b** $\frac{1}{5}$ **c** -2.50

10 **a** $\frac{\sqrt{5}}{2}$

b $\frac{a + 3\mathrm{i}}{2 + a\mathrm{i}} = \frac{5a}{4 + a^2} + \frac{-a^2 + 6}{4 + a^2}\mathrm{i}$,
for $\arg z = \frac{\pi}{4}$ real and imaginary parts must be equal
$\Rightarrow a^2 + 5a - 6 = 0$
$\Rightarrow a = -6$ or 1
a cannot be negative otherwise $\arg z$ is negative
$\therefore a = 1$

11 **a** $z_1 = \sqrt{2}\left(\cos\left(-\frac{3\pi}{4}\right) + \mathrm{i}\sin\left(-\frac{3\pi}{4}\right)\right)$ and
$z_2 = 2\left(\cos\left(\frac{\pi}{3}\right) + \mathrm{i}\sin\left(\frac{\pi}{3}\right)\right)$

b **i** $2\sqrt{2}$ **ii** $\frac{\sqrt{2}}{2}$

c **i** $-\frac{5\pi}{12}$ **ii** $\frac{11\pi}{12}$

12 **a** $|z| = |2 - 2\mathrm{i}\sqrt{3}| = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{16} = 4$

b $\arg z = -\frac{\pi}{3}$

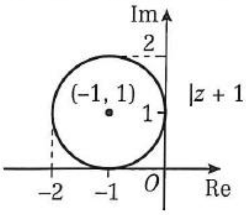
c $\left|\frac{w}{z}\right| = 1$

d $\arg\left(\frac{w}{z}\right) = \frac{\pi}{12}$

13 $4\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$

14 a $(x+1)^2 + (y-1)^2 = 1$

b



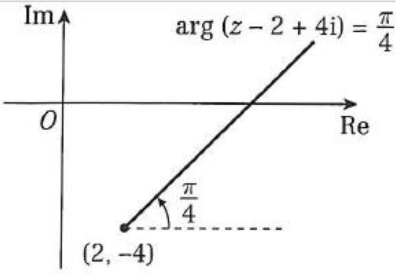
c $|z|_{\min} = \sqrt{2} - 1$

$|z|_{\max} = \sqrt{2} + 1$

d $|z-1|_{\min} = \sqrt{5} - 1$

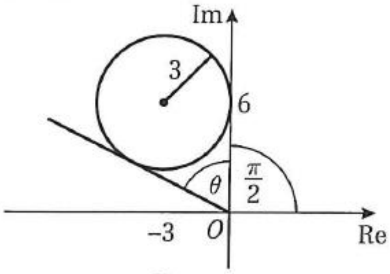
$|z-1|_{\max} = \sqrt{5} + 1$

15 a



b $3\sqrt{2}$

16

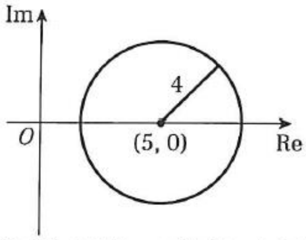


Max value $= \frac{\pi}{2} + \theta$

$$\sin\left(\frac{\theta}{2}\right) = \frac{3}{\sqrt{3^2 + 6^2}} = \frac{3}{\sqrt{45}} = \frac{1}{\sqrt{5}}$$

$$\Rightarrow \frac{\pi}{2} + \theta = \frac{\pi}{2} + 2\arcsin\left(\frac{1}{\sqrt{5}}\right)$$

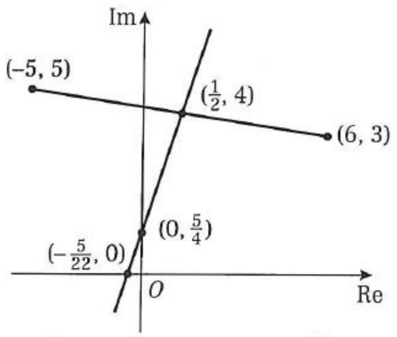
17 a



b (3.96, 3.86) and (1.14, −1.03)

c $-\pi < \theta < -0.41,\ 0.41 < \theta < \pi$

18 a



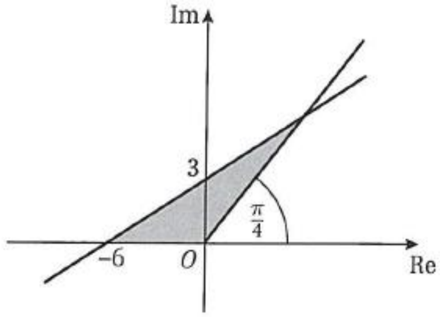
b $y = \frac{11}{2}x + \frac{5}{4}$

c $\frac{\sqrt{5}}{10}$

19 a $y = \frac{1}{2}x + 3$

b $6 + 6i$

c



20 a i $y = x - 2$

ii $(x-2)^2 + y^2 = 8$

b $-2i,\ 4 + 2i$

c